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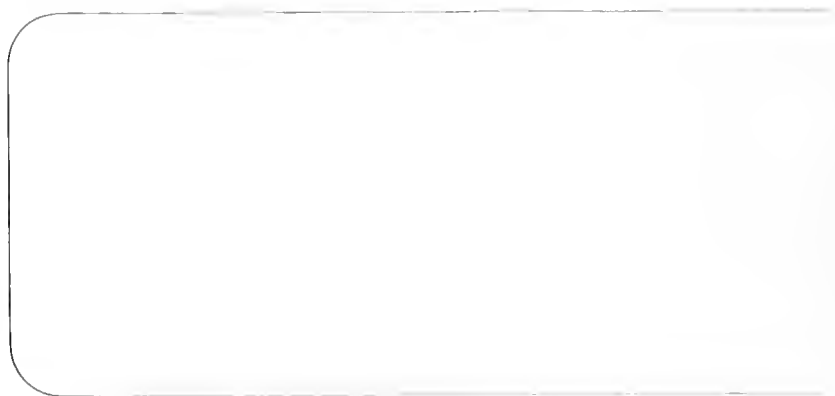
Faculty Working Papers

**ON THE ESTIMATION OF
LAGGED EFFECTS OF ADVERTISING**

Johny K. Johansson

#73

**College of Commerce and Business Administration
University of Illinois at Urbana-Champaign**



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QUESTION 1

Consider the following two functions $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$
$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

QUESTION 1(a) (10 marks)

Find the limit $\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 0} g(x)$.

Answer: $\lim_{x \rightarrow 0} f(x) = 0$ and $\lim_{x \rightarrow 0} g(x) = 0$.

The lagged effect of media advertising upon market share is estimated using standard methods. Results indicate no lagged effect. The use of a more flexible lag structure, polynomial distributed lags, shows, however, that a considerable lagged effect might exist.

When lagged effects of one (independent) variable upon another (dependent) variable are estimated, one of two standard methods is usually employed. Under the first method, lagged values of the independent variable are explicitly introduced and their coefficient values estimated without restrictions. In order to establish the correct length of the lagged effects, the estimates are derived for successively longer lags, and an appropriate cutoff point determined. Usually this occurs where coefficients go to zero or change signs (see Ferber & Verdoorn, 1962, p. 345 or Griliches, 1967, p. 30). Under the second method, restrictions are imposed a priori upon the coefficients of the lagged variables, usually in the form of some specific functional relationship between successive coefficients. Although a number of different distributions of coefficients have been suggested (see Ferber & Verdoorn, 1962, pp. 338-352, and Malinvaud, 1966, Chapter 15), by far the most commonly used is the exponential decline first proposed by Koyck (1954). As is well known, it entails the introduction of the dependent variable lagged one period as one explanatory variable in addition to the contemporary value of the original independent variable.

Both of these approaches have their drawbacks. The first, empirical, approach has no theoretical justification. In addition, successive values of the independent variable will often be closely related creating problems

of multicollinearity. The Koyck approach has an attractive interpretation in that early effects are higher and later effects gradually taper off, which would correctly describe a number of observed phenomena (see Dhrymes, 1971, Chapter 2). In addition, it limits multicollinearity and preserves degrees of freedom by reducing the number of parameters to be estimated and by eliminating the need for introducing long lags explicitly in the estimating equations. On the other hand, ordinary least squares estimates of the coefficients are not unbiased. Unless very severe assumptions on the errors are satisfied, they are not even consistent (see Johnston, 1963, p. 217). Because of the difficulty of developing satisfactory alternatives, however, most applied work still uses the ordinary least squares techniques (see, for example, Palda, 1964, and Simon, 1969).

In what follows an application of these two approaches to the estimation of the lagged effect of media advertising upon brand purchases is presented and analyzed. Although the result seems clearcut (no lagged effect), it is contrary to expectations based upon other work. Accordingly, it is judged desirable to evaluate the possibility of lagged effects using a very flexible form of the lag structure. Polynomial distributed lags are shown to be very promising in this respect. After discussing this distributed lag approach in some detail, it is shown that a constrained quadratic form explains and predicts the data fairly well. On the basis of this result it is argued that a lagged effect of advertising might very well obtain, and that, accordingly, the standard methods for testing the existence of a lagged effect are not conclusive.

The Data

The advertising data used consist of monthly brand expenditures for 4 large national brands within a non-seasonal consumer product class, with an average purchase frequency of once a month. These expenditures are broken down into three different media (Network TV, Spot TV, and Magazines). In addition, Newspaper data are available in the form of brand lineage figures but only for 6 months' aggregates. Monthly Newspaper observations thus consist of interpolated figures.¹ The four brands together represent about 30% of the total market.

The purchase data come from repeated monthly surveys of product users' brand purchases. In addition to the basic purchase information, data on preferences, trial rates, degree of usage, and number of deal purchases are included. The monthly sample size in general number about 600 respondents. A total of 13 consecutive months for each brand is available.² Although the four national brands dealt with here all are well established, consecutive months show considerable advertising variations as well as variations in purchase share. It seems therefore feasible to use this product class for a study of the short run effects of media advertising.

¹This leads, of course, to the existence of measurement error in the Newspaper term with unfortunate results on its coefficient estimate. It was still judged desirable to keep the variable in so as not to ascribe possible Newspaper effects erroneously to other media. The Newspaper coefficient estimates should be interpreted very conservatively, however.

²A more extensive discussion of the data base can be found in Johansson (1972), Chapter 4.

Model Specification

The dependent variable, purchase or market share of a brand, was measured as the proportion of product users who responded that they bought that brand last time.³ So as not to ascribe to media advertising effects that emanated from other sources, the deal, trial, and preference proportions in each sample were introduced as explanatory variables in the estimating models. Because of forced correlations--if a respondent had purchased the brand last time he/she would also be classified as a trier, for example--the preference and trial variables were lagged one period.

The early regression runs were aimed at specifying the models more precisely. With the low number of observations available for each brand, it was deemed very desirable to pool the observations in some fashion. As the brands were all national and well established, it seemed justifiable to assume the coefficients for the trial, deal, and preference variables to be very similar. With reference to the media advertising variables, however, the basic heterogeneity of the variables (due to creative, vehicle, and other within-media-differences) forestalled a parallel argument. In the Koyck model such differences would also make for different coefficients on the lagged dependent variable.

Thus, the initial runs allowed for separate coefficients for each brand's media variables (and lagged dependent variable when the Koyck specification was used), but constrained the trial, deal, and preference coefficients to be the same for each brand.⁴ Although the low degrees of

³This measure of market share is not the one usually encountered and should be seen as a "proxy" variable.

⁴The approach used assigns dummy variables for each brand's separate slope coefficient; it is well discussed by Gujarati (1970).

freedom made tests of the significance of the coefficients weak, the results were quite consistent. The advertising coefficients were low and very similar across brands.

Because of the low number of observations (with a lagged dependent variable introduced, there were 12 data points per brand) only current monthly advertising (4 media variables) was included in these runs. In order to assess differences between brands for lagged advertising, all media advertising for the preceding month was summed and included as one additional variable. Again, no great brand differences emerged in terms of the advertising coefficients. Runs were made in parallel for advertising expenditures and for advertising shares (where the denominator consisted of the four brands' total advertising in the particular medium). No differences between brands obtained, although shares tended to do better (in terms of signs and significance levels of the coefficient estimates) than expenditures. When different functional forms (linear, semi-logarithmic, and double-logarithmic versions) were run, the double-logarithmic one generally exhibited the higher R-squares, and was the one kept for the later runs. In terms of differences between brands, however, no consistent distinctions emerged. Finally, running heavy and light users separately uncovered no significant differences (except for the intercept) between the two groups; as all the survey data consistently was broken down into these two user categories, it was clear that heavy and light users might be pooled.

For a few early runs, the media expenditures were aggregated into total advertising also for the current month, and in another few cases two media--total TV, and total print media--variables were used. In terms of differences

between brands the results were very much as before. For the final runs the four media split was maintained. It was deemed desirable to allow different lags for different media to appear; additionally, in the short run media might show differential effects (see, for example, Bogart, 1967).

For the final runs, then, the four brands were pooled allowing only a separate intercept (but no separate slope coefficients) for each brand, in the usual analysis of covariance approach. With the lagging of the dependent variable one period, there were 12 observations per brand; with four brands pooled, there were 48 observations. In addition, the heavy and light users were pooled, again allowing for a separate intercept. Thus, the number of observations available for the final runs amounted to 96.⁵

Results I

Before the final regression runs were started, it was decided that with 96 observations it would be possible to keep some data points out of the estimations of the alternative lag models, making a test of each model's predictive power possible. True, in the earlier model specification runs all observations available had been used, so that strictly speaking no observations were "untouched." On the other hand, these final runs would test alternative lag structures against each other, and this question had not been at issue in these earlier runs. Rather, the early runs had served so as to specify the characteristics of the models that were to stay maintained throughout all alternative lag versions. These characteristics comprised the functional form (double-logarithmic), the use of advertising

⁵Because the advertising data were collected independently of the monthly surveys, the restriction to 13 months total time span did not apply to the advertising data. As will be seen later, this facilitated greatly the application of the polynomial distributed lags.

shares instead of expenditures, the four-way media split, the specification of the non-advertising variables, the pooling approach used but not the lag to be used. Thus, a test comparing the predictive performance of the alternative models was judged feasible, and to this end 16 arbitrarily chosen data points were eliminated before the final runs.⁶

The first of the final regression runs was made for the Koyck specification with contemporaneous advertising and a lagged dependent variable introduced on the right hand side. The results from this run are depicted in the first row of coefficients in Table 1. As can be seen, the lagged dependent variable's coefficient is highly insignificant indicating that there is no exponentially declining lagged effect. Of the media variables, only Network TV is seen to have a significant positive effect at the .05 level, with Spot TV closest to significance of the rest of the media. The other coefficients are largely as expected--as no particular hypotheses concerning them were developed for the purpose of the present research they will generally be ignored in what follows.

In order to see whether the insignificance of the lagged dependent variable could be due to multicollinearity it was deleted in a second run. The results are depicted in the second coefficient row of Table 1. As we see, the standard errors of all the coefficients as well as the coefficients themselves stay remarkably stable, perhaps with the exception of the trial variable. Judging from the R^2 the introduction of the lagged dependent

⁶The number of observations that were eliminated was judgmentally determined on the basis of number of parameters to be estimated, the total number of observations, and the desire to get enough observations for a discrimination of predictive performance. As the usual procedure in time series data is to predict the last observations available, the 16 data points were picked from the last two months of each brand, for both heavy and light users.

variable does not increase the explanation. We might as well describe the variations in the dependent variable without it.

Before letting go completely of the Koyck hypothesis, however, it was decided to give it another chance by introducing media advertising lagged one period in addition to the contemporaneous advertising variables. By leaving in this way the last period's media coefficients unrestricted we allow for the possibility of the maximum impact of advertising occurring at $t-1$ instead of t , the exponential decline setting in only after that.⁷ The results of this run are displayed in the third row of Table 1. As could perhaps have been expected after the first results, the lagged dependent variable is still highly insignificant and very close to zero.⁸ In addition, the media inputs of the previous period have negative, although insignificant coefficients. The exception is Newspaper advertising, but here the current period's advertising coefficient is now negative. Considering the interpolated data points, the implied multicollinearity of successive Newspaper advertising is hardly surprising. As we can see from the R^2 (unadjusted for degrees of freedom), the introduction of these 4 additional variables in fact produces very little improvement in the goodness of the fit.⁹

⁷See, for example, Griliches, 1967, p. 24.

⁸The low value of the lagged dependent variable is probably partly due to the fact that the brands were well established in the market.

⁹This run can also be seen as a misspecification test for autocorrelation (see Griliches, 1967, p. 34). The results indicate that the autocorrelation model is not appropriate, as the negative media coefficients at $t-1$ are insignificant (at the .05 level).

Finally, in line with the empirical approach to lagged effects, this last run was repeated with the lagged dependent variable omitted. The result is shown in the fourth row of Table 1. Again as expected, the omission makes little difference. All in all, judging from the negative sign of the media coefficients, the correct structure does not include the previous period's media advertising. Applying the criterion of simplicity, the second equation seems the best one, indicating no lagged effects of media advertising.

Another Approach: Polynomial Distributed Lags

Because this result was so unexpected, it was decided that some other lag structures should also be investigated.¹⁰ Perhaps a longer lag would be appropriate despite the results of the fourth run if some lag structure different from the exponential decline could be developed? It was decided that the polynomial distributed lag approach developed by Almon (1965) would possibly provide such a structure, allowing the distribution of coefficients to be described by a polynomial function of appropriate degree. Because this approach is fairly new, a brief description of the method seems necessary.

Write the distributed lag model in its general form¹¹

$$(1) \quad Y_t = b_0 X_t + b_1 X_{t-1} + \dots + b_{p-1} X_{t-p+1}$$

where Y_t and X_t represent time series observations and the b_i , $i=0,1,\dots,p-1$, are the p coefficients of the lag function. Under the Koyck hypothesis

¹⁰In principle, such an investigation would be worthwhile even though results were according to expectations. In practice, of course, the investigation is often halted before that.

¹¹The present discussion of the approach, although equivalent to the Almon (1965) treatment, is drawn from the reformulated version presented by Hall (1967).

the b_i are related to each other by

$$(2) \quad b_i = bk^i, \quad 0 < k < 1, \quad i=0,1,\dots,$$

The basic hypothesis underlying the polynomial distributed lag approach is that the lag distribution b_i is a smooth function of the lag i . This smooth function, in turn, can be approximated closely by a polynomial of fairly low order, that is

$$(3) \quad b_i = a_1 + a_2 i + a_3 i^2 + \dots + a_N i^{N-1}, \quad i = 0,1,\dots,p-1,$$

where N would usually be less than 6. It is immediately seen that the assumption (3) is much less restrictive than the hypothesis (2), which limits the coefficients to an exponential decline. As we will see, the estimation problems plaguing the Koyck model are also resolved in the polynomial approach. First, substitute equation (3) into (1):

$$(4) \quad Y_t = \sum_{i=0}^{p-1} (a_1 + a_2 i + \dots + a_N i^{N-1}) X_{t-i}$$

$$= a_1 \left(\sum_{i=0}^{p-1} X_{t-i} \right) + a_2 \left(\sum_{i=0}^{p-1} i X_{t-i} \right) + \dots + a_N \left(\sum_{i=0}^{p-1} i^{N-1} X_{t-i} \right).$$

Then we define new variables, $z_{t,j}$, which are moving averages of the original variables, such that

$$(5) \quad z_{t,j} = \sum_{i=0}^{p-1} i^{j-1} X_{t-i}, \quad j=1,\dots,N.$$

Substituting the new variables (5) into (4) we have a linear model of the ordinary form

$$(6) \quad Y_t = a_1 z_{t,1} + a_2 z_{t,2} + \dots + a_N z_{t,N}$$

All estimation methods which are appropriate for linear equations are available for estimation of distributed lags if the method of polynomial approximation is used.

As can be seen from equation (6), the number of parameters to be estimated is equal to N , i.e., one more than the degree of the polynomial. In estimation, the investigator first settles upon the (trial) degree of the polynomial, then calculates the new variables given by (5), and finally estimates equation (6). Different degrees of the polynomial can be tried, selecting the value of N on the basis of goodness of fit measures. Similarly, different lengths of the total lag period can be tried out in the cases where the exact value of p is uncertain.

Because the z -variables used in the regression are linear combinations of the original variables (see equation (5)), the sampling theory becomes very straightforward. The original coefficients b_i are simply linear combinations of the estimated coefficients a_j , and the standard errors are then derived easily (see Goldberger, 1964, p. 167). A derivation is presented in the Appendix.

Finding the Best Lag Structure

It was decided first to investigate the length of the total lag period. To this end, a quadratic lag was estimated for p equal to 4, 5, 6, and 7; that is, for time lags up to $t-3$, $t-4$, $t-5$, and $t-6$, respectively.¹² The results were not very encouraging, in that more than half of the newly constructed z -variables did not enter the equations for the lags up to $t-5$ and $t-6$ (using the program's minimum cutoff level). With a quadratic

¹²The quadratic form is simple; furthermore, it goes well with the notion that advertising effect over time has a sigmoid shape (Rao, 1970, p. 66).

lag applied to each of the four media, there were three coefficients (a_1 , a_2 , and a_3) to be estimated for each medium, making for 12 advertising variables, and the collinearities between the variables turned out to be very high. The z-variables did enter the t-4 equation, which thus looked more promising. But no media coefficients were significant, a result that also obtained for the t-3 equation. As the longer lag gave more flexibility, it was decided to continue with the t-4 form.

Next a cubic lag structure was investigated. No clear improvement in the goodness of the fit or the significance of the media coefficients were registered, however, and instead of attempting an even higher degree polynomial lag, another approach was taken. It was decided to return to the quadratic function and fit a restricted version of it. These constraints were derived from two quite reasonable hypotheses. First, it seemed logical to assume that advertising in the next, coming, time period would have no impact upon the present purchases. Second, judging from our results so far, one could conjecture that advertising had no effect after four months (t-4). Incorporating these two constraints into the fitted function was done through equation (3), setting b_{-1} and b_{t-5} equal to zero and solving for the z's. We have the first constraint for N=3:

$$(7) \quad b_{-1} = a_1 + a_2(-1) + a_3(-1)^2 = 0;$$

and the second constraint

$$(8) \quad b_5 = a_1 + a_2(5) + a_3(5)^2 = 0;$$

the solutions for a_2 and a_3 in terms of a_1 are easily derived as

$$(9) \quad a_2 = -\frac{4}{5} a_1 \text{ and } a_3 = -\frac{1}{5} a_1.$$

These values are then substituted into equation (6) so that, with N=3, we have

$$(10) \quad Y_t = a_1 z_{t,1} + \frac{4}{5} a_1 z_{t,2} - \frac{1}{5} a_1 z_{t,3} \quad .$$

Moving a_1 out from each term on the right hand side, rewriting the z-variables in terms of their original X-components, and combining like terms, we have

$$(11) \quad Y_t = a_1 (X_t + \frac{8}{5} X_{t-1} + \frac{9}{5} X_{t-2} + \frac{8}{5} X_{t-3} + X_{t-4}) = a_1 z_{t,0} \quad .$$

Thus the introduction of two constraints has reduced the number of coefficients to be estimated by two, leaving only one parameter (per medium) to be estimated for the quadratic form. From the estimate of this parameter a_1 we can then derive the estimates of a_2 and a_3 using the relations (9), and then finally obtain the original coefficients b_i from equation (3).

Because of the pattern of the weighting coefficients in (9) this constrained quadratic lag structure becomes completely symmetric between $t+1$ and $t-5$. With a_1 positive, the lag will attain a maximum at $t-2$. With the shape predetermined, the estimate of a_1 only serves to determine the height of this "humped" lag structure. In Diagram 1 the constrained quadratic lag is depicted together with the two versions of the exponential decay model estimated earlier.

Results II

The result from the constrained quadratic lag run is presented in the last row of Table 1. Considering the earlier results, the estimates are surprisingly good: Network TV advertising is again significant (at the .05 level), and so is Newspaper advertising (although, again, the measurement problem in that variable should be accounted for). The R-square

is very similar to the earlier models presented.¹³ The one drawback of this quadratic fit relative to the earlier ones is the negativity of the trial coefficient--because of its insignificance (at the .05 level) and less than central concern for the study it was disregarded, however.

The coefficients in Table 2 indicate the symmetry of this particular form of the lag structure, with a peak effect at period $t-2$. As was indicated above, if the effect of advertising over time exhibits an initial threshold, a takeoff, and then a saturation, this is the type of structure we would expect the lag coefficients to show. Although basically empirically derived, this particular structure thus has a reasonable theoretical rationalization.

Other combinations of degree of polynomial and length of lag could have been investigated--a constrained cubic was in fact fitted for which some of the media coefficients turned out negative. Also, different lags for different media could have been tried--a constrained quadratic form with a peak at $t-3$ was estimated for magazine advertising, but the outcome was similar to the earlier results. On the whole, however, the constrained quadratic structure found was deemed sufficiently good to challenge the no lagged effects of the second regression run, and no additional runs were made.

The five alternative models were then compared in terms of their predictive ability on the remaining 16 observations. The values of the independent variables were introduced for each observation, and a predicted value of the dependent variable generated. In Diagram 2 the predictions

¹³This kind of stability could be a sign that the variables that are changed (in this case the media variables) have no impact on the dependent variable. The significant media coefficients belie that. In addition, the beta coefficients showed that the advertising impact was considerable for all the five models reported here (although, as one would expect, the deal variable and the user constant had a greater impact).

from the five models plus the actual values of the dependent variable are displayed and Table 3 gives the same information in deviation form. As can be seen, although the predictions are generally not very good, the agreement between the predicted and actual values is somewhat better for the models with advertising at $t-1$ introduced (Models 3 and 4) than for those without these terms (Models 1 and 2). The best fit, however, seems to be given by the constrained quadratic lag model (Model 5). To further check on these results, the correlation between predicted and actual values was computed and a "predictive" R-square derived. These predictive R-squares are displayed in the last row of Table 3. The results are confirmed: The constrained quadratic lag structure does a better job of prediction than the alternative four structures.

Discussion

As we have seen, the evidence presented seems to favor a "humped" advertising lag for all four media with a peak after two months over the exponential decline hypotheses investigated. This result is somewhat different from most lag structures uncovered in advertising research. Palda (1964), Simon (1969), Bass and Parsons (1969), Montgomery and Silk (1972), and Bass and Clarke (1972) all found support for the exponential decay. In the case of Palda and Simon, however, the data are yearly and thus the concern is with more long run effects. The Bass and Parsons study uses bimonthly observations which means that "current" advertising is relatively aggregated and containing at least last month's advertising. Also, their different results might be due to product differences. The Montgomery and Silk article finds current month's journal advertising highest, although the exponential decline does not set in until $t-6$. In this case, the discrepancies could possibly be explained through audience differences: Montgomery and Silk deal with ethical drugs, and the

advertising receiver will generally be the doctor prescribing the drug, rather than the ultimate consumer. Finally, in the case of Bass and Clarke, their best lag structure peaks at $t-1$, with a subsequent exponential decay. The difference this time may be due to frequency of purchase differences; the Bass and Clarke product being bought weekly in the majority of cases, compared to our product's monthly purchases. Thus, the differences in results between our study and these studies can perhaps be explained as fairly natural.

Turning back to the present study, can we really say then that the "humped" lag structure in fact does hold for this product class? The answer is "not necessarily." Even though the predictive ability of the constrained quadratic seems somewhat better, it is still not very good. There might certainly exist some other lag structure whose predictive performance is better. Furthermore, the proposed best structure was found after considerable search in the data, and although it has a nice theoretical interpretation as resulting from the S-curve, a rigorous test could be done only after new data were collected.

For the present research, this model choice question is perhaps not so crucial, however. What is shown through the results in Table 1 is that in fact the common approach to determining the appropriate lag structure is not conclusive. It is not only that the Koyck structure might be inappropriate--the very specific assumption behind that approach obviously will not fit every case. But in addition it is clear that the empirical approach which introduces successively longer lags has pitfalls as well. What is needed in the case where the exact lag structure is not well known a priori is a sufficiently rich family of structures that can be compared empirically before a final choice is made. The polynomial

distributed lags used here might very well be preferable in this respect to the simple empirical approaches often used.

APPENDIX

Sampling Theory For The Polynomial Distributed Lags Coefficients

Define z_t to be an N -component row vector for the left hand side of (5). Similarly, let x_t denote a p -component row vector for the X_{t-i} variables on the right hand side of (5). Finally, let B denote the p by N matrix of i^{j-1} - coefficients in (5). Then we can write (5) as

$$(12) \quad z_t = x_t B \quad .$$

After estimating the a_j we can get the estimates of the b_i via

$$(13) \quad \hat{b} = B\hat{a} \quad ,$$

where \hat{b} is a p by 1 vector of the b_i estimates, and \hat{a} is an N by 1 vector of the regression estimates of the a_j . This is equivalent to inserting the estimated a_j into (3) and computing the b_i from that equality. Then the variance-covariance matrix $V(\hat{b})$ can be computed as

$$(14) \quad V(\hat{b}) = B V(\hat{a}) B' \quad ,$$

from which the standard errors are directly obtainable as the square roots of the diagonal elements.

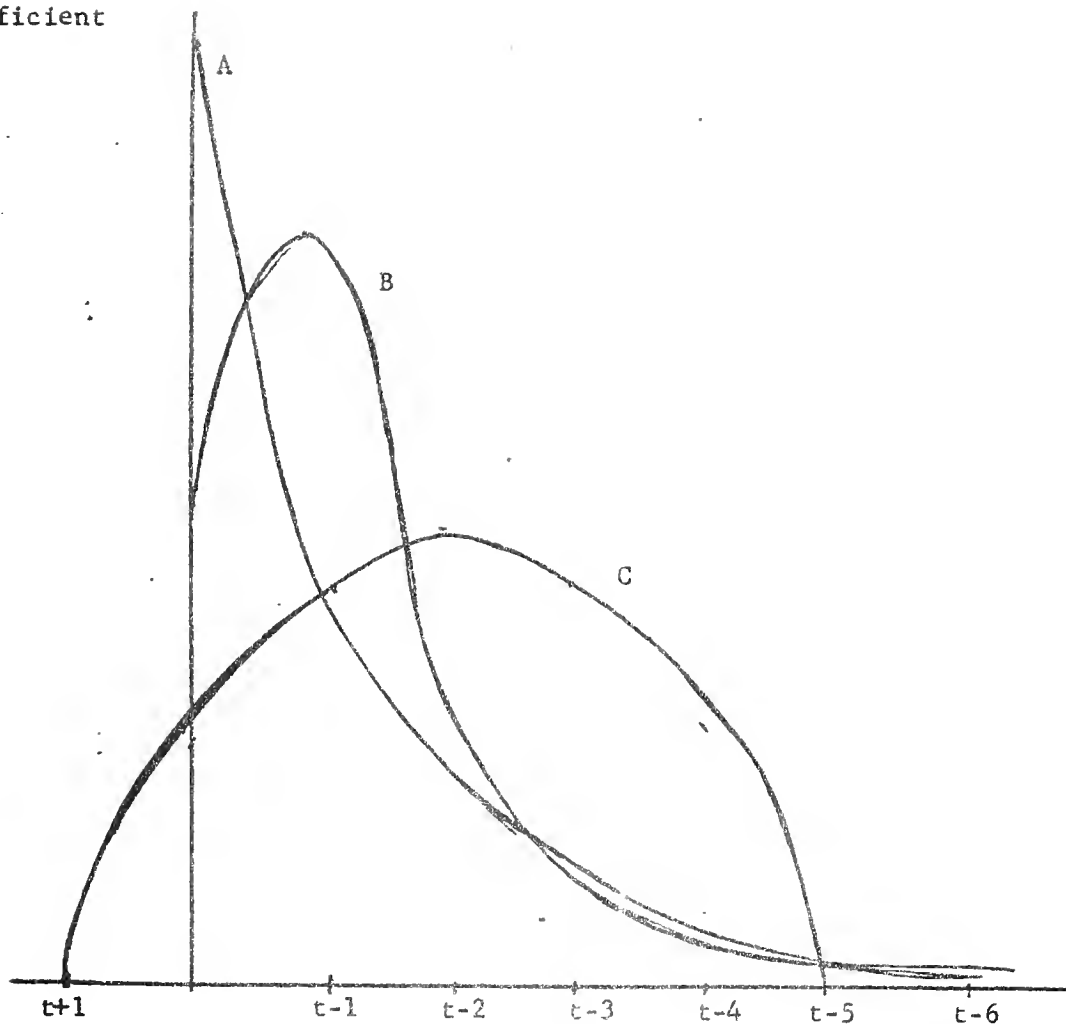
To get the total impact over time of the independent variable we sum the individual b_i . Letting u denote a p by 1 vector of ones, we get the sum s as

$$(15) \quad s = u' B \hat{a} \quad ,$$

and $V(s)$ becomes

$$(16) \quad V(s) = u' B V(\hat{a}) B' u \quad .$$

Coefficient



Lag

DIAGRAM 1: Three different lag structures

- A: Exponential Decay, with greatest impact for current advertising (KOYCK)
- B: Exponential Decay, with greatest impact for advertising lagged one period
- C: Constrained Quadratic Lag Structure.

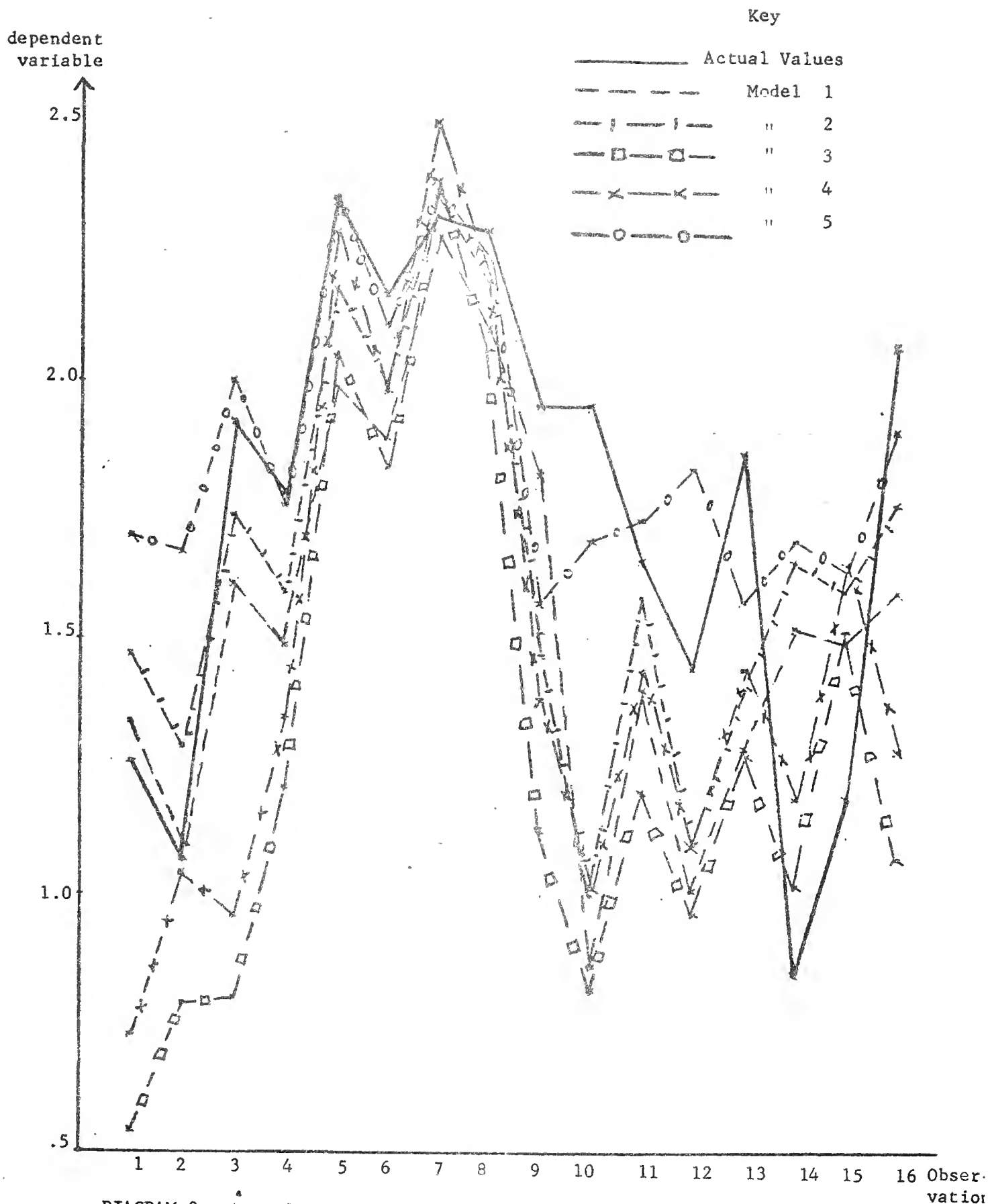


DIAGRAM 2: Actual vs. Predicted Values for the 16 Left-out Observations

(Note: The 16 observations do not represent consecutive months, but rather 2 months per brand and usage group -- see main text.)

TABLE 1: REGRESSION COEFFICIENTS AND STANDARD ERRORS

Independent Variable: # Purchasers of Brand $t = \text{Pur } t$
 # Users of Product t
 Functional Form: Doublelogarithmic
 Advertising is in Shares: One brand's \$ in a medium
 All 4 brands' \$ in the medium
 Observations = 96 - 16 = 80

Coefficients significant at the .05 level are marked by *.

INDEPENDENT VARIABLES														
NEWS t	MAG t-1	NET TV t-1	SPOT TV t-1	NEWS t-1	PUR t-1	DEAL t-1	PREF. t-1	TRIAL t-1	CONSTANTS					Unad- justed R ²
									OVER- ALL	BRAND 1	BRAND 2	BRAND 3	HEAVY- LIGHT USERS	
.002 (.016)					-.046 (.199)	.381* (.113)	.065 (.143)	.208 (.196)	.752* (.260)	-.195 (.294)	.003 (.192)	-.088 (.150)	-.505* (.128)	.64
.002 (.016)						.380* (.112)	.050 (.126)	.181 (.157)	.765* (.252)	-.206 (.288)	-.005 (.187)	-.092 (.148)	-.495* (.121)	.64
-.026 (.027)	-.008 (.022)	-.039 (.029)	-.039 (.033)	.022 (.026)	-.001 (.206)	.427* (.125)	.031 (.145)	.252 (.200)	.707* (.278)	-.014 (.368)	.016 (.193)	-.099 (.169)	-.548* (.135)	.66
-.026 (.027)	-.008 (.021)	-.039 (.028)	-.039 (.032)	.022 (.026)		.427* (.124)	.031 (.126)	.252 (.162)	.707* (.273)	-.014 (.365)	.016 (.189)	-.099 (.165)	-.548* (.128)	.66
NEWS TOTAL ¹														
.056* (.032)						.344* (.114)	.152 (.121)	-.270 (.179)	1.657* (.411)	-1.026 (.898)	-.033 (.309)	-.155 (.317)	-.370* (.121)	.64

TABLE 1: REGRESSION COEFFICIENTS AND STANDARD ERRORS

Dependent Variable: $\frac{\# \text{ Purchasers of Brand } t}{\# \text{ Users of Product } t}$ $t = \text{Pur}_t$

Functional Form: Doublelogarithmic

Advertising is in Shares: One brand's \$ in a medium

of Observations = 96 - 16 = 80

MAG = Magazine Advertising
 NET TV = Network TV Advertising
 SPOT TV = Spot TV Advertising
 NEWS = Newspaper Advertising
 DEAL = # On Deal Purchaser/# Users
 PREF. = # Preferring Brand/# Users
 TRIAL = # Having Tried Brand/# Users

Coefficients significant at the .05 level are marked by *.

MODEL	INDEPENDENT VARIABLES										CONSTANTS					Unad-justed R ²			
	MAG t	NET TV t	SPOT TV t	NEWS t	MAG t-1	NET TV t-1	SPOT TV t-1	NEWS t-1	PUR t-1	DEAL t-1	PREF. t-1	TRIAL t-1	OVER-ALL	BRAND 1	BRAND 2		BRAND 3	HEAVY-LIGHT USERS	
1	.007 (.020)	.079* (.024)	.015 (.026)	.002 (.016)					-.046 (.199)	.381* (.113)	.065 (.143)	.208 (.196)	.752* (.260)	-.195 (.294)	.003 (.192)	-.088 (.150)		-.505* (.128)	.64
2	.007 (.020)	.079* (.024)	.016 (.025)	.002 (.016)						.380* (.112)	.050 (.126)	.181 (.157)	.765* (.252)	-.206 (.288)	-.005 (.187)	-.092 (.148)		-.495* (.121)	.64
3	.004 (.020)	.071* (.025)	.036 (.029)	-.026 (.027)	-.008 (.022)	-.039 (.029)	-.039 (.033)	.022 (.026)	-.001 (.206)	.427* (.125)	.031 (.145)	.252 (.200)	.707* (.278)	-.014 (.368)	.016 (.193)	-.099 (.169)		-.548* (.135)	.66
4	.004 (.020)	.071* (.024)	.036 (.029)	-.026 (.027)	-.008 (.021)	-.039 (.028)	-.039 (.032)	.022 (.026)		.427* (.124)	.031 (.126)	.252 (.162)	.707* (.273)	-.014 (.365)	.016 (.189)	-.099 (.165)		-.548* (.128)	.66
5	.056 (.067)	.140* (.052)	.091 (.091)	.056* (.032)						.344* (.114)	.152 (.121)	-.270 (.179)	1.657* (.411)	-1.026 (.898)	-.033 (.309)	-.155 (.317)		-.370* (.121)	.64

¹See Table 2.

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MODEL	MAG t	NET TV t	SPOT TV t
1	.007 (.020)	.079* (.024)	.015 (.026)
2	.007 (.020)	.079* (.024)	.016 (.025)
3	.004 (.020)	.071* (.025)	.036 (.029)
4	.004 (.020)	.071* (.024)	.036 (.029)
5	MAG TOTAL .056 (.067)	NET TV TOTAL .140* (.052)	SPOT TV TOTAL .091 (.091)

¹See Table 2.

TABLE 2: LAGGED MEDIA COEFFICIENTS FROM POLYNOMIAL RUN¹

	MAG	NET TV	SPOT TV	NEWS
t	.008	.020	.013	.008
t - 1	.013	.032	.021	.013
t - 2	.014	.036	.023	.014
t - 3	.013	.032	.021	.013
t - 4	.008	.020	.013	.008
TOTAL	.056 (.067)	.140* (.052)	.091 (.091)	.056* (.032)

¹The significance level of the coefficients is the same as that of the total impact coefficient (see Appendix). The symmetry of the lag structure is seen directly; as indicated in the text, only one parameter per medium was estimated.

TABLE 3

Predictive Ability of the 5 Alternative Models Over
the 16 Left-Out Observations

Brand	Observa- tion No.	Actual Values	Deviations = Predicted-Actual Values				
			Model				
			1	2	3	4	5
1	1	1.26	.08	.21	-.73	-.53	.44
	2	1.07	.03	.22	-.29	-.03	.60
	3	1.92	-.31	-.18	-1.12	-.96	.08
	4	1.78	-.29	-.19	-.56	-.43	-.02
2	5	2.35	-.36	-.17	-.30	-.06	.01
	6	2.17	-.28	-.18	-.34	-.19	-.06
	7	2.32	-.04	.04	.06	.18	.05
	8	2.29	-.19	-.11	-.23	-.11	-.05
3	9	1.95	-.63	-.44	-.82	-.57	-.38
	10	1.95	-1.09	-.95	-1.14	-.94	-.24
	11	1.65	-.25	-.07	-.45	-.22	.08
	12	1.44	-.43	-.34	-.48	-.34	.39
4	13	1.86	-.58	-.46	-.59	-.42	-.29
	14	.85	.67	.80	.17	.34	.84
	15	1.19	.30	.40	.32	.45	.44
	16	2.07	-.48	-.31	-1.00	-.79	-.17
Predic- tive R ²			.33	.34	.40	.41	.48

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